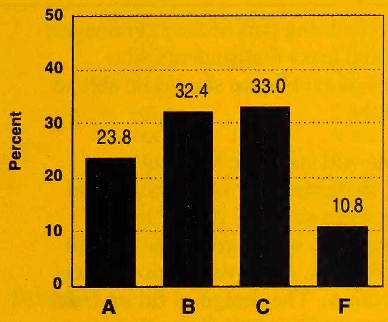
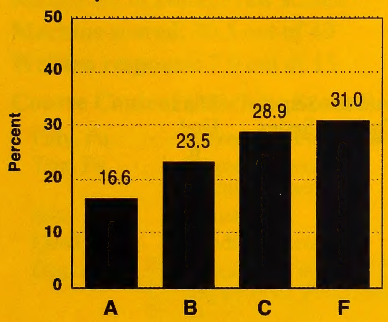
Mathematics 30

Diploma Examination Results
Examiners' Report for June 1996

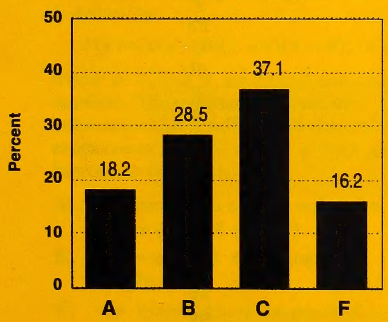
School-Awarded Mark



Diploma Examination Mark



Final Course Mark



The summary information in this report provides teachers, school administrators, students, and the general public with an overview of results from the June 1996 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used with the detailed school and jurisdiction reports that have been mailed to schools and school jurisdiction offices. An annual provincial report containing a detailed analysis of the combined January, June, and August results is published each year.

Description of the Examination

The Mathematics 30 Diploma Examination consists of three parts: a multiple-choice section of 40 questions and a numerical-response section of nine questions, together worth 70%, and a written-response section of three questions, worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final course marks achieved by 9 342 students who wrote the June 1996 examination.

- 83.8% of the 9 342 students achieved the acceptable standard (a final course mark of 50% or higher).
- 18.2% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 50% of the students who wrote the June 1996 examination were females.

- 83.6% of the female population achieved the acceptable standard (a final course mark of 50% or higher).
- 16.9% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 50% of the students who wrote the June 1996 examination were males.

- 84.0% of the male population achieved the acceptable standard (a final course mark of 50% or higher).
- 19.5% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Students First!
Student Evaluation

Alberta
EDUCATION

Provincial Averages

- The average school-awarded mark was 67.0%.
- The average diploma examination mark was 59.5%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 63.6%.

Of the 9 342 students who wrote the June 1996 examination, 1 739 had written at least one Math 30 exam previously.

Results and Examiners' Comments

This examination has a balance of question types and difficulties reflecting the philosophy of the Mathematics 30 Course of Studies. It was designed so that students who are achieving the acceptable standard in Mathematics 30 should obtain a mark of 50% or higher. Students who are achieving the standard of excellence in Mathematics 30 should obtain a mark of 80% or higher. Students who are achieving the acceptable standard or the standard of excellence are expected to be able to achieve the curriculum standards identified in the *Mathematics 30 Information Bulletin, Diploma Examination Program*. At least 70% of the examination includes questions and tasks that students who achieve the acceptable standard should be able to complete successfully. The remaining part of the examination includes questions and tasks that students who achieve the standard of excellence should be able to complete successfully.

Future examinations will continue to focus on assessing students' understanding of mathematical concepts and on problem solving. Students will continue to be expected to solve problems, explain solutions, justify solutions, or prove solutions in the written-response section. The design of future field tests and examinations will include items that assess how well students have achieved the general learner expectations stated in the Mathematics 30 Course of Studies.

Blueprint

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC 1	A	0.841	√							P
MC 2	A	0.673	√							PS
MC 3	D	0.391	√							C
MC 4	C	0.777	√							C
MC 5	B	0.490	√							P
MC 6	D	0.724	√							PS
MC 7	A	0.526	√							PS
MC 8	D	0.644		√						C
MC 9	B	0.657		√						P
MC 10	A	0.666		√						C
MC 11	C	0.512		√						PS
MC 12	A	0.547		√						P
MC 13	C	0.407		√						PS
MC 14	B	0.571					√			C
MC 15	D	0.599					√			P
MC 16	C	0.795					√			P
MC 17	C	0.652					√			P
MC 18	A	0.741					√			PS
MC 19	D	0.520					√			PS
MC 20	B	0.427					√			P
MC 21	B	0.827				√				C
MC 22	C	0.505				√				C
MC 23	C	0.600				√				C
MC 24	A	0.576				√				C
MC 25	B	0.360				√				PS
MC 26	B	0.563				√				PS
MC 27	C	0.766							√	P
MC 28	D	0.747							√	C
MC 29	B	0.704							√	P
MC 30	A	0.607							√	P
MC 31	D	0.733							√	P
MC 32	C	0.796							√	C

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC 33	A	0.834						√		PS
MC 34	B	0.781						√		PS
MC 35	A	0.630						√		C
MC 36	A	0.859						√		PS
MC 37	D	0.505						√		C
MC 38	B	0.700			√					P
MC 39	B	0.696			√					P
MC 40	C	0.708			√					C
NR 1	23.3	0.666	√							P
NR 2	10.5	0.363		√						C
NR 3	233	0.463		√						P
NR 4	160	0.647					√			PS
NR 5	75.4	0.626							√	P
NR 6	3.6	0.440				√				C
NR 7	0.20	0.468						√		PS
NR 8	16	0.611						√		PS
NR 9	7.5	0.609			√					P
WR 1	—	0.557								PCPS
WR 2	—	0.503								PCPS
WR 3	—	0.514								PCPS

Subtest

When analyzing detailed results, please bear in mind that subtest results **cannot** be directly compared.

Results are in average raw scores.

Machine scored: 30.5 out of 49

Written response: 7.9 out of 15

Course Content (Machine Scored)

Poly. Fn.	Polynomial Functions	5.1 out of 8
Trig. Fn.	Trigonometric and Circular Functions	4.3 out of 8
Stat.	Statistics	2.7 out of 4
Quad. Rltns.	Quadratic Relations	3.9 out of 7
Exp. & Log.	Exponential and Logarithmic Functions	5.0 out of 8
Perm. & Com.	Permutations and Combinations	4.7 out of 7
Seq. & Series	Sequences and Series	5.0 out of 7

Mathematical Understandings*

- Procedural (P): 11.6 out of 18
- Conceptual (C): 9.7 out of 16
- Problem Solving (PS): 9.2 out of 15

*Refer to Appendix D of the 1995–96 *Mathematics 30 Information Bulletin, Diploma Examinations Program,* for an explanation of mathematical understandings. These are the mathematical abilities described in Appendix G.

Multiple-Choice and Numerical-Response Questions

The multiple-choice and numerical-response sections of the examination ask a sample of questions that cover all content areas in Mathematics 30. A discussion about how well students achieve the curriculum standards in the units of Polynomial Functions and Quadratic Relations follows.

Polynomial Functions — In order to achieve the curriculum standards for this unit, students should be able to recognize and give examples of polynomial functions of different degrees, generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages, use the Remainder Theorem to evaluate a third-degree integral polynomial for rational values of the variable, and understand how these values can be used to find factors of the polynomial function. In addition, students should be able to factor and find the zeros for an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational. Students are required

3. A family of third-degree polynomial functions is defined by

$$P(x) = a(x - b)(x - c)(x - d), \quad a \neq 0$$

where b, c, and d are distinct non-zero real numbers. The **minimum** amount of information that is sufficient to determine the exact values of a, b, c, and d for a specific function in this family is

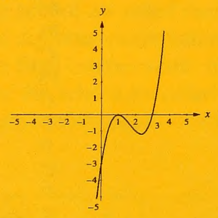
A. the coordinates of three points that lie on the graph of $y = P(x)$

B. the y-intercept of the graph of $y = P(x)$

C. the x-intercepts of the graph of $y = P(x)$

•D. both the x-intercepts and y-intercept of the graph of $y = P(x)$

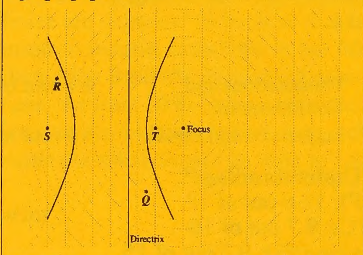
The graph of $y = (x-1)^2(x-3)$ is shown below.



1. If $A(4.7, m)$ is a point on the graph, then the value of m, correct to the nearest tenth, is ___________ .
(Record your answer on the answer sheet.)

Answer: **23.3**

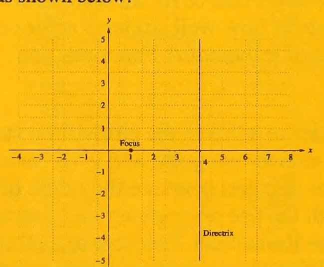
A hyperbola and the points Q, R, S, and T are shown on the circle-line graph paper below.



24. One of the points, Q, R, S, or T, lies on a hyperbola that has an eccentricity greater than the eccentricity of the hyperbola shown. If the focus and directrix are the same for both hyperbolas, then the point that satisfies this condition is point

•**A.** Q
B. R
C. S
D. T

The focus and the directrix of a quadratic relation are $(1, 0)$ and $x = 4$, as shown below.



26. If a point $(b, 0)$ lies on this quadratic relation, then a parabola is formed when

A. $b = 1$
•**B.** $b = 2.5$
C. $1 < b < 2.5$
D. $2.5 < b < 4$

to find approximations for all the real zeros of integral polynomial functions using graphing calculators or computers, derive an equation of an integral third-degree polynomial function, given its rational zeros, and be able to recognize the general shape of graphs of integral polynomial functions of degree 4 or less where the multiplicity of zeros is one, two, or three. Also, students should be able to identify the potential rational zeros of an integral polynomial function, determine the minimum degree of a polynomial function by using the multiplicities of its zeros, and participate in and contribute toward the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30. Multiple-choice questions 1 to 7 and numerical-response question 1 required students to demonstrate their understanding of this unit. Of the students who achieved the acceptable standard but not the standard of excellence, 87.4% answered multiple-choice question 1 correctly, 71.1% answered multiple-choice question 2 correctly, 37.1% answered multiple-choice question 3 correctly, 82.7% answered multiple-choice question 4 correctly, 46.7% answered multiple-choice question 5 correctly, 73.8% answered multiple-choice question 6 correctly, 55.1% answered multiple-choice question 7 correctly, and 72.2% answered numerical-response question 1 correctly. Multiple-choice questions 1, 4, 6, and 7 and numerical-response question 1 identified expectations for those students who achieved the acceptable standard but not the standard of excellence; over 70% of students who achieved the acceptable standard on the examination were able to successfully achieve the expectations of these questions.

Students who achieve the standard of excellence are expected to be able to use the Remainder Theorem when either the factor or the original polynomial contains unknown coefficients, and also be able to use this theorem to evaluate integral polynomial functions beyond the third degree for rational values of the variable, and to understand how this can be used to find factors of the polynomial function. In addition, these students can derive an equation for an integral polynomial function, given its zeros and any other information that will uniquely define it, as well as recognize the general shape of graphs of integral polynomial functions of degree n where the multiplicity of zeros is greater than two. Standard of excellence achievement also requires students to complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30. Multiple-choice questions 2, 3, and 5 required students to show that they can do this. Of the students who achieved the standard of excellence on the examination, 90.7% answered multiple-choice question 2 correctly, 68.1% answered multiple-choice question 3 correctly, and 76.0% answered multiple-choice question 5 correctly.

Quadratic Relations — To achieve the acceptable standard in quadratic relations, students must be able to describe orally, in writing, and by modeling, each of the following: the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle. They must also be able to identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola. Students

must be able to describe orally and in writing each of the following: the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$; state the quadratic relation formed when given the value of the eccentricity; state the eccentricity when given the quadratic relation; state the quadratic relation formed when given the locus definition; and state the effects on the graph of the quadratic relation in the form

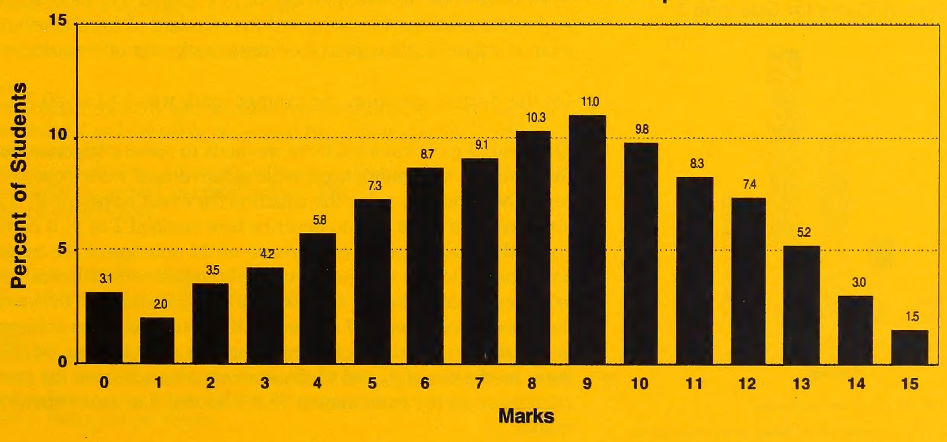
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when TWO of the numerical coefficients change. Students are also expected to generate the graphs of quadratic relations with the use of graphing calculators or a graphing utility package; identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity; calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the quadratic relation; and identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line. Multiple-choice questions 21 to 26 and numerical-response question 6 required students to demonstrate their understanding of this unit. Of the students who achieved the acceptable standard but not the standard of excellence, 87.4% correctly answered multiple-choice question 21; 51.2% correctly answered multiple-choice question 22; 64.2% correctly answered multiple-choice question 23; 60.6% correctly answered multiple-choice question 24; 33.1% correctly answered multiple-choice question 25; 61.4% correctly answered multiple-choice question 26; and 46.0% correctly answered numerical-response question 6. In addition to the expectations for the acceptable standard, students who achieve the standard of excellence must also be able to identify and to describe orally, in writing, and by modeling, the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola; the changes in the graph of a quadratic relation when the eccentricity changes; the locus definition and use it to verify the equation of each conic section; the effects on the graph of the quadratic relation in the form

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change; and the solution to problems that require the analysis of quadratic relations studied in Mathematics 30. Multiple-choice question 26 requires this of students. Of the students who achieved the standard of excellence, 92.3% answered question 26 correctly.

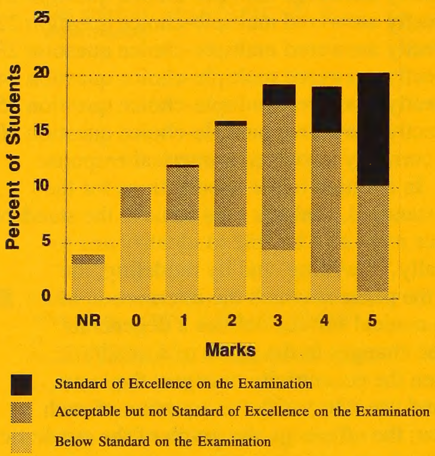
Written-Response Question

As published in the 1994–95 and 1995–96 *Mathematics 30, Diploma Examination Information Bulletins*, the written-response questions assess whether or not students can draw on their mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions do not necessarily fall into a particular unit of study but may cross more than one unit or may require students to make connections among mathematical concepts. Students achieving the acceptable standard were expected to obtain at least half marks on all questions. Students achieving the standard of excellence were expected to answer all questions almost perfectly.

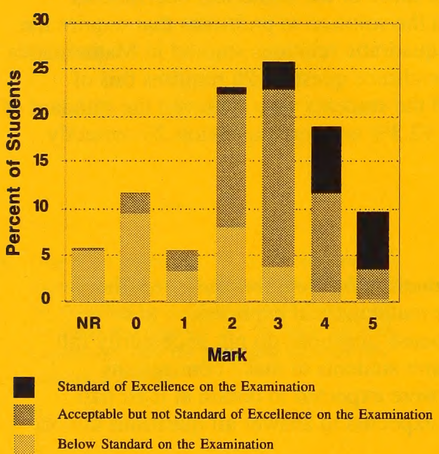
Distribution of Marks for Written Response



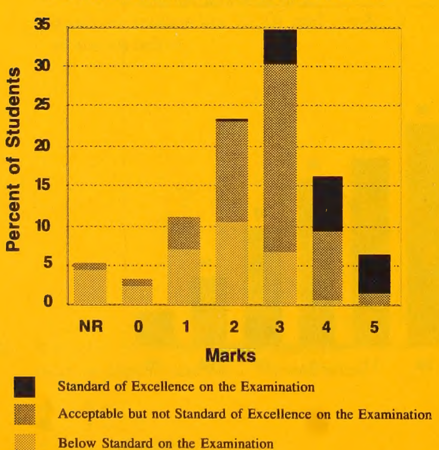
Distribution of Marks for Question 1



Distribution of Marks for Question 2



Distribution of Marks for Question 3



Question 1 required students to clearly explain why there is no difference in the number of ways of assigning rooms to 5 people or assigning rooms to 6 people, if the owner of a small motel had 6 motel rooms to rent, where each person could only rent one room. Students were also asked to find the value of t_k where t_k is clearly defined, for given values of k, and finally, to generalize the number of ways the owner can assign k people to m rooms. Students who achieve the acceptable standard in Mathematics 30 are expected to participate in the problem-solving process for problems involving permutations and/or combinations, including probability problems. Students who achieve the standard of excellence are expected to complete the solution to such problems. Students who achieved the acceptable standard were expected to state that 6! = 720 and use some model to explain why the number of ways of assigning 6 people to 6 hotel rooms is 6!. However, these students had no explanation or a very weak explanation for the number of ways to assign 5 people to 6 rooms. These students would have been able to determine the value for t_k for $1 \leq k \leq 6$ but would not have been able to formulate the generalization required in the third part of the question. It was expected that students achieving this standard of achievement would score 3 out of 5 marks. Of the students who met the acceptable standard of achievement on the examination, 66.9% received at least 3 out of 5 marks. Students who achieved the standard of excellence on the examination were expected to score 4 or 5 marks out of 5 marks.

On this 5-mark question, the average mark was 2.78 or 55.7%.

Question 2 required students to algebraically or arithmetically show that two sums of an arithmetic sequence were equal, given the first term of the sequence, the common difference, and a graphical representation of the sum of the terms of the arithmetic sequence. Students were also asked to provide an example of arithmetic sequence for which the sum of the first k terms was **never** equal to the sum of the first j terms and, finally, to identify and explain conditions for an arithmetic sequence to have $S_k = S_j$ for some $j = k, j, \ k \in N$. Students achieving the acceptable standard are expected to algebraically or arithmetically show that the two sums of a given arithmetic sequence were equal and also be able to provide an example of an arithmetic sequence where this condition would never be true. Students achieving the acceptable standard of achievement were expected to score 3 or more marks out of 5, and students achieving the standard of excellence were expected to score 4 or more marks out of 5 marks. Of the students who achieved the acceptable standard of achievement on the examination, 62.6% scored 3 or more marks out of 5, and of the students who achieved the standard of excellence on the examination, 78.3% scored 4 or more marks out of 5 marks.

On this 5-mark question, the average mark was 2.51 or 50.3%.

Question 3 gave students three methods to solve a trigonometric equation. Students demonstrating acceptable achievement were expected to be able to use Method 1 to solve the equation for exact values of θ. They were also expected to be able to describe how method 2 or 3, if carried out correctly, would lead to an *estimate* of the solution of the trigonometric equation. Students who achieved the standard of excellence were also expected to describe the significance of the distinction between the use of the words *determine* and *estimate*. Of the students who achieved the acceptable standard of achievement on the examination, 64.5% scored 3 or more marks out of 5, and of the students who achieved the standard of excellence on the examination 58.6% scored 4 or more marks out of 5 marks.

On this 5-mark question, the average mark was 2.57 or 51.4%

Scoring Guide for Written-Response Questions

Credit may be given to students who show unusual insight. If their solutions fall outside *Specific Question Scoring Rubrics*, they will be scored against the *General Scoring Guide* shown below.

GENERAL SCORING GUIDE

5 marks The student
- demonstrated a *complete understanding* of the problem
- used mathematical knowledge and problem-solving techniques to find the solution
- justified the solution and explained its relevance to the problem

4 marks The student
- demonstrated *an understanding* of the problem
- chose a strategy that used mathematical knowledge and problem-solving techniques to find a solution, but the procedure contained *a minor flaw*
- showed *some justification* of his or her results

3 marks The student
- demonstrated *some understanding* of the problem
- formulated *some aspects* of the problem mathematically
- demonstrated the use of a strategy that used mathematical knowledge and problem-solving techniques to find a *partial* solution
- communicated little understanding of the complexities of the problem

2 marks The student
- explored the *initial stages* of the problem
- applied *some* relevant mathematical knowledge and problem-solving techniques to find a *partial* solution

1 mark The student
- applied some relevant mathematical knowledge to the problem

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SPECIFIC QUESTION SCORING RUBRICS

Question 1

5 The student
- explained why there is no difference in the number of ways of assigning 5 or 6 people to 6 rooms
- determined the value of t_k for $1 \le k \le 6$ by calculating the values of t_k to be 6, 30, 120, 360, 720, 720 **OR**

  $t_k = \frac{6!}{(6-k)!}$ *(or equivalent form)*
- determined that the number of ways a motel owner assigns k people to m rooms is $t_k = \frac{m!}{(m-k)!}$ *(or equivalent form)*

4 The student
- explained why there is no difference in the number of ways of assigning 5 or 6 people to 6 rooms **and** determined the value of t_k for $1 \le k \le 6$ by calculating the values of t_k to be 6, 30, 120, 360, 720, 720 *(or equivalent form)*

  OR $t_k = \frac{6!}{(6-k)!}$ *(or equivalent form)* **OR**

  determined the number of ways a motel owner assigns k people to m rooms is $t_k = \frac{m!}{(m-k)!}$

3 The student
- did the first bullet correctly and *attempted* one or both of the next two bullets **OR**
- *attempted* bullet 1 and did either of the next two bullets correctly **OR**
- did not attempt bullet 1 and
  1) correctly did one of the other two bullets and *attempted* the other bullet **OR**
  2) correctly did the second and third bullets **OR**
- attempted all three bullets

2 The student
- attempted two of the three bullets **OR**
- correctly answered any one of the three bullets

1 The student
- attempted to explain why there are 720 ways to assign 5 people to 6 motel rooms by labelling the rooms and the people and recognizing the arrangements are permutations. The explanation is not clear. **OR**
- attempted to find t_k, $1 \le k \le 4$ **OR**
- attempted to find t_k for m rooms

Question 2

5 The student
- showed $S_2 = 15$ and $S_5 = 15$ **and** provided an example of an arithmetic sequence/series where $S_k \neq S_j$ **and** demonstrated an understanding of the problem by identifying and explaining conditions necessary for an arithmetic sequence to have $S_j = S_k$ for some $j \neq k,\ j,\ k \in N$.

4 The student
- showed $S_2 = 15$ and $S_5 = 15$ **and** provided an example of an arithmetic sequence/series where $S_k \neq S_j$ **and** demonstrated an understanding of the problem by identifying and explaining conditions for an arithmetic sequence to have $S_j = S_k$ for some $j \neq k,\ j,\ k \in N\ n \neq k$, but the explanation contained a minor flaw.

3 The student
- showed $S_2 = 15$ and $S_5 = 15$
- provided an example of an arithmetic sequence/series where $S_k \neq S_j$, etc. **OR**
- provided a sufficient explanation for $S_j = S_k$, $j \neq k$ **OR**
- showed $S_2 = 15$ and $S_5 = 15$ **and** attempted an explanation for $S_j = S_k$, $j \neq k$

2 The student
- showed $S_2 = 15$ and $S_5 = 15$ **OR**
- attempted to show $S_2 = S_5$ **and** provided an example of an arithmetic sequence/series where $S_j \neq S_k$ **OR**
- provided an example where $S_j \neq S_k$, and attempted to provide conditions for $S_j = S_k$

1 The student
- attempted to determine $S_2 = S_5 = 15$ **OR**
- provided an example of an arithmetic sequence/series where $S_k \neq S_j$ **OR**
- attempted to provide conditions for $S_j = S_k$,

Question 3

5 The student
- determined the solution algebraically
- clearly communicated why the word *determine* could be used in **Method 1** but the word *estimate* is used in **methods 2** and **3**.
- described how **method 2** or **3** leads to the solution

4 The student
- determined the solution to the equation algebraically **and** correctly answered one of the other two bullets **OR**
- attempted the algebraic solution (with a minor flaw) **and** correctly answered the other two bullets

3 The student
- determined the solution algebraically **OR**
- clearly communicated how **method 2 or 3** leads to the solution **and** clearly explains the difference in the use of the words *determine* and *estimate* **OR**
- explored the initial stages of the algebraic solution **and** clearly communicated how **method 2 or 3** leads to the solution **OR** clearly explained the difference in the use of the words *determine* and *estimate*

2 The student
- explored the initial stages of the algebraic solution **OR**
- clearly communicated why the word *determine* could be used in **Method 1** but the word *estimate* is used in **methods 2** and **3** **OR**
- communicated how **method 2** or **3** leads to the solution

1 The student
- commented on one method. The response is not clearly communicated, but the reader can determine that the student knows that the method would work **OR**
- attempted to explain the difference in the use of the words *determine* and *estimate* **OR**
- makes a correct substiution for $\sin^2\theta$, but no other work is shown

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For further information, contact Marion Florence (mflorence@edc.gov.ab.ca) or Phill Campbell (pcampbell@edc.gov.ab.ca) at the Student Evaluation Branch at 427-0010. To call toll-free from outside of Edmonton, dial 310-0000.